Neodređeni integral

1. Primitivna f-ja i neodređeni integral. Osnovne formule integriranja.

Određivanje f-je $F(x)$ iz datog diferencijala $dF(x)=f(x)\,dx$ (ili iz neke date derivacije $F'(x)=f(x)$) nazivamo integriranje, a traženu f-ju $F(x)$ nazivamo primitivna f-ja f-je $f(x)$. Drugim riječima efekat suprotan diferenciranju nazivamo integriranje.

Navedimo nekoliko primjera primitivnih f-ja:

- $F(x)=\cos x$ je primitivna f-ja f-je $f(x)=\sin x$ zato što je

  $F'(x)=f(x)$ $\quad ((\cos x)'=\sin x)$ ili

  $dF(x)=f(x)\,dx$ $\quad (d(\cos x)=\sin x\,dx)$

- $F(x)=\frac{1}{4}x^4$ je primitivna f-ja f-je $f(x)=x^3$ zato što je

  $F'(x)=f(x)$ $\quad \left(\left(\frac{1}{4}x^4\right)'=\frac{1}{4}\cdot 4x^3=x^3\right)$ ili

  $dF(x)=f(x)\,dx$ $\quad \left(d\left(\frac{1}{4}x^4\right)=\frac{1}{4}d(x^4)=\frac{1}{4}\cdot 4x^3dx=x^3dx\right)$

- $F(x)=\operatorname{tg} x$ je primitivna f-ja f-je $f(x)=\frac{1}{\cos^2 x}$ zato što je

  $F'(x)=f(x)$ $\quad \left((\operatorname{tg} x)'=\frac{1}{\cos^2 x}\right)$ ili

  $dF(x)=f(x)\,dx$ $\quad \left(d(\operatorname{tg} x)=\frac{1}{\cos^2 x}dx\right)$.

- $F(x)=\arcsin x$ je primitivna f-ja f-je $f(x)=\frac{1}{\sqrt{1-x^2}}$ zato što je

  $dF(x)=f(x)\,dx$ $\quad \left(d(\arcsin x)=\frac{1}{\sqrt{1-x^2}}dx\right)$ ili

  $F'(x)=f(x)$ $\quad \left((\arcsin x)'=\frac{1}{\sqrt{1-x^2}}\right)$.

- $F(x)=\ln|x|$ je primitivna f-ja f-je $f(x)=\frac{1}{x}$.

  ZA VJEŽBU OBJASNITI ZAŠTO.

Svaka neprekidna f-ja f-je $f(x)$ ima beskonačno mnogo različitih primitivnih f-ja, koje se jedna od druge razlikuju u članu koji predstavlja konstantu:
ako je $F(x)$ primitivna f-ja f-je $f(x)$ (tj. ako je $F'(x)=f(x)$) tada je i $F(x)+c$ primitivna f-ja od $f(x)$, gdje je c proizvoljna konstanta. Zašto? Zato što

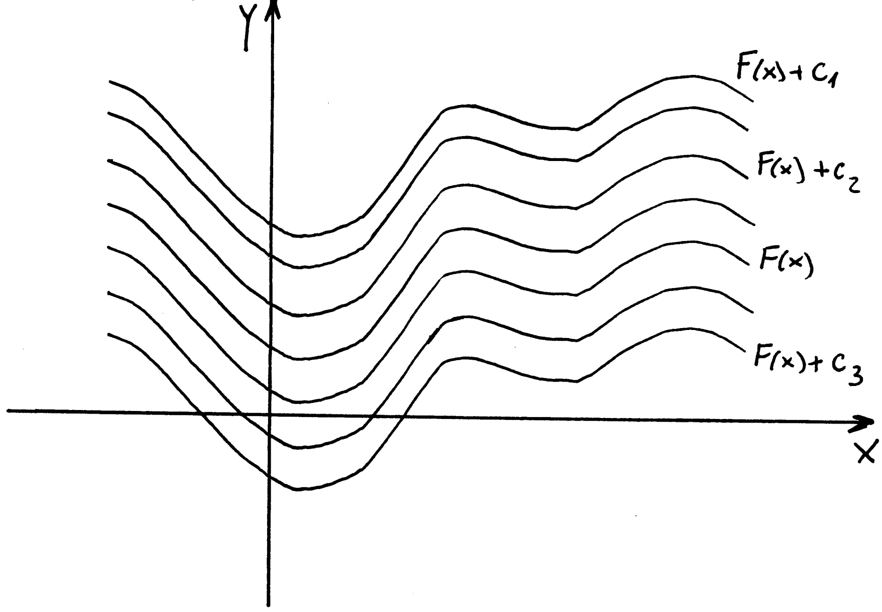
$$(F(x)+c)' = F'(x) = f(x).$$

Opšti izraz $F(x)+c$ skupa svih primitivnih f-ja f-je $f(x)$ zovemo <u>neodređeni integral</u> f-je $f(x)$ i označavamo ga sa znakom $\int$:

$$\int f(x)\,dx = F(x) + c \qquad \text{akko} \qquad d\left[F(x)+c\right] = f(x)\,dx$$

(ako i samo ako)

Geometriski, u xOy koordinatnom sistemu, grafici svih primitivnih f-ja date f-je $f(x)$ predstavljaju familiju krivih, koje zavise od parametra c, i koje se mogu izvesti jedna iz druge paralelnom translacijom duž y-ose.



Osobine neodređenog integrala:

I $\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x)$ ili $d\int f(x)\,dx = f(x)\,dx$

(izvod integrala) (diferencijal integrala)

II $\int F'(x)\,dx = F(x) + c$ ili $\int dF(x) = F(x) + c$

III $\int a\,f(x)\,dx = a\int f(x)\,dx$ tj. konstantu a koja množi f-ju možemo izvesti ispred znaka integrala

IV $\int [f_1(x) + f_2(x) - f_3(x)]\,dx = \int f_1(x)\,dx + \int f_2(x)\,dx - \int f_3(x)\,dx$

tj. integral sume je jednak sumi integrala svih članova

Osnovne formule integriranja:

1. $\int u^a\,du = \frac{u^{a+1}}{a+1} + c, \quad a \neq -1$

2. $\int u^{-1}\,du = \int \frac{du}{u} = \ln|u| + c$

3. $\int a^u\,du = \frac{a^u}{\ln a} + c$

$\int e^u\,du = e^u + c$

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4. $\int \sin u\,du = -\cos u + c$

5. $\int \cos u\,du = \sin u + c$

6. $\int \frac{du}{\cos^2 u} = tg\,u + c$

7. $\int \frac{du}{\sin^2 u} = -ctg\,u + c$

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8. $\int \frac{du}{u^2 + a^2} = \frac{1}{a}\,arc\,tg\,\frac{u}{a} + c$

9. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + c$

10. $\int \frac{du}{\sqrt{a^2 - u^2}} = arc\,sin\,\frac{u}{a} + c$

11. $\int \frac{du}{\sqrt{u^2 + a}} = \ln\left|u + \sqrt{u^2 + a}\right| + c$

U osnovnim formulama integriranja a predstavlja konstantu, u je nezavisna promjenjiva ili bilo koja (diferencijabilna) f-ja neke nezavisne promjenjive. Navedimo nekoliko primjera korištenja osnovnih formula integriranja:

- Integral $I_1 = \int \sqrt{x}\, dx = \int x^{\frac{1}{2}} dx$ predstavlja formulu 1 $\left(\int u^a du = \frac{u^{a+1}}{a+1} + c,\ a \neq -1\right)$ gdje su $u = x$, $a = \frac{1}{2}$.
Prema toj formuli $I_1 = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}\sqrt{x^3} + c$.

- Integral $I_2 = \int 3^x dx$ predstavlja formulu 3 $\left(\int a^u du = \frac{a^u}{\ln a} + c\right)$ gdje su $a = 3$, $u = x$. Prema toj formuli $I_2 = \frac{3^x}{\ln 3} + c$.

- Integral $I_3 = \int \frac{dt}{t^2+3}$ predstavlja formulu 8 $\left(\int \frac{du}{u^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{u}{a} + c\right)$ gdje su $u = t$, $a = \sqrt{3}$. Prema toj formuli $I_3 = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{t}{\sqrt{3}} + c$.

- Integral $I_4 = \int \frac{d\varphi}{\sqrt{\varphi^2 - 5}}$ predstavlja formulu 11 $\left(\int \frac{du}{\sqrt{u^2+a}} = \ln\left|u + \sqrt{u^2+a}\right| + c\right)$ gdje su $u = \varphi$, $a = -5$. Prema toj formuli $I_4 = \ln\left|\varphi + \sqrt{\varphi^2 - 5}\right| + c$

- Integral $I_5 = \int \frac{2x}{x^2+7} dx = \int \frac{(x^2+7)'}{x^2+7} dx = \int \frac{d(x^2+7)}{x^2+7}$ predstavlja formulu 2 $\left(\int \frac{du}{u} = \ln|u| + c\right)$ pri čemu je $u = x^2+7$ (zato što je $d(x^2+7) = 2x\,dx$).
Prema toj formuli $I_5 = \ln(x^2+7) + c$. Znak apsolutne vrijednosti smo izostavili zato što je uvijek $x^2+7 > 0$.

U opštem slučaju, u formulama 2, 9 i 11

$$\left(\int \frac{du}{u} = \ln|u| + c, \quad \int \frac{du}{u^2-a^2} = \frac{1}{2a} \ln\left|\frac{u-a}{u+a}\right| + c \quad i \quad \int \frac{du}{\sqrt{u^2+a}} = \ln\left|u+\sqrt{u^2+a}\right| + c \right)$$

pišemo apsolutnu vrijednost samo u slučaju kada izraz ispod logaritma može imati negativnu vrijednost.

- Integral $I_6 = \int 5 \sin 5t \, dt = \int \sin 5t \, d(5t)$ predstavlja formulu 4 $\left(\int \sin u \, du = -\cos u + c \right)$ pri čemu je $u = 5t$. Prema toj formuli

$$I_6 = -\cos 5t + c.$$

- Integral $I_7 = \int e^{\sin\varphi} \cos\varphi \, d\varphi = \int e^{\sin\varphi} d\sin\varphi$ (zato što $d\sin\varphi = \cos\varphi \, d\varphi$) predstavlja formulu 3 $\left(\int e^u du = e^u + c \right)$, pri čemu je $u = \sin\varphi$. Možemo zaključiti $I_7 = e^{\sin\varphi} + c$.

- Posmatrajmo integral $I_8 = \int \frac{e^x dx}{e^{2x}-1}$. Primjetimo da, zato što je $de^x = e^x dx$ možemo pisati $I_8 = \int \frac{e^x dx}{e^{2x}-1} = \int \frac{de^x}{(e^x)^2-1}$. Prema formuli 9 $\left(\int \frac{du}{u^2-a^2} = \frac{1}{2a} \ln\left|\frac{u-a}{u-a}\right| \right)$ pri čemu je $u = e^x$, $a = 1$ možemo zaključiti $I_8 = \frac{1}{2} \ln \frac{|e^x-1|}{e^x+1} + c$.

(#) Naći sljedeće integrale i provjeriti rezultat diferenciranjem

a) $\int \frac{dx}{x^3}$ b) $\int \frac{dx}{\sqrt{2-x^2}}$ c) $\int 3^t 5^t \, dt$ d) $\int \sqrt{y+1}\, dy$

e) $\int \frac{dx}{2x^2-6}$

Rj: a) $\int \frac{dx}{x^3} = \int x^{-3} dx = \frac{x^{-2}}{-2} + C = C - \frac{1}{2x^2}$

Koristili smo formulu $\int u^\alpha du = \frac{u^{\alpha+1}}{\alpha+1} + C$

gdje je $u = x$, $\alpha = -3$.

Provjera:

Diferenciramo dobijenu f-ju

$$d\left(C - \frac{1}{2x^2}\right) = -\frac{1}{2}(x^{-2})' dx = \left(-\frac{1}{2}\right)(-2)\, x^{-3} dx = x^{-3} dx$$

$$= \frac{dx}{x^3}$$

b) $\int \frac{dx}{\sqrt{2-x^2}} = \arcsin\frac{x}{\sqrt{2}} + C$

Koristili smo formulu

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

gdje je $u = x$, $a = \sqrt{2}$

Provjera:

$d\left(\arcsin\frac{x}{\sqrt{2}} + C\right) =$

$$=\left(\arcsin\frac{x}{\sqrt{2}}\right)' dx = \frac{\left(\frac{x}{\sqrt{2}}\right)'}{\sqrt{1-\left(\frac{x}{\sqrt{2}}\right)^2}} dx = \frac{dx}{\sqrt{2-x^2}}.$$

c) $$\int 3^t 5^t dt = \int 15^t = \frac{15^t}{\ln 15} + c$$

Koristili smo formulu $\int a^u du = \frac{a^u}{\ln a} + c$ pri čemu je $a = 15$, $u = t$

Provjera: $$d\left(\frac{15^t}{\ln 15} + c\right) = \frac{1}{\ln 15}(15^t)' dt = \frac{1}{\ln 15} 15^t \ln 15 \, dt$$
$$= 15^t dt$$

d) $$\int \sqrt{y+1}\, dy = \int (y+1)^{\frac{1}{2}} d(y+1) = \frac{(y+1)^{\frac{3}{2}}}{\frac{3}{2}} + c =$$
$$= \frac{2}{3}\sqrt{(y+1)^3} + c$$

Koristili smo formulu $\int u^\alpha du = \frac{u^{\alpha+1}}{\alpha+1} + c$ pri čemu je $u = y+1$, $\alpha = \frac{1}{2}$ $(d(y+1) = dy)$

Provjera:
$$d\left(\frac{2}{3}\sqrt{(y+1)^3} + c\right) = \frac{2}{3}\left(\sqrt{(y+1)^3}\right)' dy = \frac{2}{3}\cdot\frac{3}{2}(y+1)^{\frac{3}{2}-1} dy$$
$$= \sqrt{y+1}\, dy$$

e) $\int \frac{dx}{2x^2-6} = \frac{1}{2}\int \frac{dx}{x^2-3} = \frac{1}{2}\cdot\frac{1}{2\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + C$

Koristili smo formulu $\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

pri čemu je $u=x$, $a=\sqrt{3}$

Provjera:

$$d\left(\frac{1}{4\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + C\right) = \frac{1}{4\sqrt{3}}\left(\ln\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)' dx$$

$$= \frac{1}{4\sqrt{3}}\left(\ln(x-\sqrt{3}) - \ln(x+\sqrt{3})\right)' dx =$$

$$= \frac{1}{4\sqrt{3}}\cdot\left(\frac{1}{x-\sqrt{3}} - \frac{1}{x+\sqrt{3}}\right) dx = \frac{1}{4\sqrt{3}}\cdot\frac{x+\sqrt{3}-x+\sqrt{3}}{x^2-3}\, dx$$

$$= \frac{dx}{2(x^2-3)}$$

(#) Odrediti integrale

a) $\int \frac{dx}{\sqrt[3]{5x}}$ b) $\int \frac{dt}{\sqrt{3-4t^2}}$ c) $\int \cos 3\varphi \, d\varphi$

d) $\int e^{-\frac{x}{2}} dx$ e) $\int \sin(ax+b) dx$ f) $\int \frac{dx}{5x+4}$

Rješenje

a) $\int \frac{dx}{\sqrt[3]{5x}} = \int (5x)^{-\frac{1}{3}} = 5^{-\frac{1}{3}} \int x^{-\frac{1}{3}} dx = \frac{1}{\sqrt[3]{5}} \cdot \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C =$

$= \frac{3}{2\sqrt[3]{5}} x^{\frac{2}{3}} + C = \frac{3}{2\sqrt[3]{5}} \sqrt[3]{x^2} + C$

Koristili smo formulu $\int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1}$ pri čemu je $u = x$, $\alpha = -\frac{1}{3}$.

b) $\int \frac{dt}{\sqrt{3-4t^2}} = \int \frac{dt}{\sqrt{4(\frac{3}{4}-t^2)}} = \frac{1}{\sqrt{4}} \int \frac{dt}{\sqrt{\frac{3}{4}-t^2}} = \frac{1}{2} \cdot \arcsin \frac{t}{\sqrt{\frac{3}{4}}} + C$

$= \frac{1}{2} \arcsin \frac{2t}{\sqrt{3}} + C$

Koristili smo folmulu $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$ gdje $u = t$, $a = \frac{\sqrt{3}}{2}$

Ovo smo mogli uraditi i na drugi način

$$\int \frac{dt}{\sqrt{3-4t^2}} = \int \frac{dt}{\sqrt{3-(2t)^2}} = \left| \begin{array}{l} d(2t) = 2\,dt \\ dt = \frac{1}{2} d(2t) \end{array} \right| = \frac{1}{2} \int \frac{d(2t)}{\sqrt{3-(2t)^2}}$$

$= \frac{1}{2} \arcsin \frac{2t}{\sqrt{3}} + C$

Koristili smo istu formulu gdje je $u = 2t$, $a = \sqrt{3}$.

c) $\int \cos 3\varphi \, d\varphi = \left| \begin{array}{l} d(3\varphi) = 3\, d\varphi \\ d\varphi = \frac{1}{3}\, d(3\varphi) \end{array} \right| = \frac{1}{3} \int \cos 3\varphi \, d(3\varphi) = \frac{1}{3} \sin 3\varphi + C$

Koristili smo formulu $\int \cos u \, du = \sin u + C$ gdje je $u = 3\varphi$.

d) $\int e^{-\frac{x}{2}} dx = \left| \begin{array}{l} d(-\frac{x}{2}) = -\frac{1}{2} dx \\ dx = -2\, d(-\frac{x}{2}) \end{array} \right| = -2 \int e^{-\frac{x}{2}} d(-\frac{x}{2}) = -2 e^{-\frac{x}{2}} + C$

Koristili smo formulu $\int e^u du = e^u + C$ gdje $u = -\frac{x}{2}$.

e) $\int \sin(ax+b)\, dx = \left| \begin{array}{l} d(ax+b) = a\, dx \\ dx = \frac{1}{a}\, d(ax+b) \end{array} \right| = \frac{1}{a} \int \sin(ax+b)\, d(ax+b) =$

$= -\frac{1}{a} \cos(ax+b) + C$

Koristili smo formulu $\int \sin u \, du = -\cos u + C$ pri čemu je $u = ax+b$

f) $\int \frac{dx}{5x+4} = \left| \begin{array}{l} d(5x+4) = 5\, dx \\ dx = \frac{1}{5}\, d(5x+4) \end{array} \right| = \frac{1}{5} \int \frac{d(5x+4)}{5x+4} = \frac{1}{5} \ln |5x+4| + C$

Koristili smo formulu $\int \frac{du}{u} = \ln|u| + C$ pri čemu je $u = 5x+4$.

(#) Odrediti integrale

a) $\int (3-2x)^7 dx$ b) $\int \frac{dx}{\cos^2(m-nx)}$ c) $\int tg\varphi \, d\varphi$

Rj. a) $\int (3-2x)^7 dx = \left| \begin{array}{l} d(3-2x) = -2\,dx \\ dx = -\frac{1}{2} d(3-2x) \end{array} \right| = -\frac{1}{2}\int (3-2x)^7 d(3-2x)$

$= -\frac{1}{2} \cdot \frac{(3-2x)^8}{8} + C = -\frac{1}{16}(3-2x)^8 + C$

Koristili smo formulu $\int u^\alpha du = \frac{x^{\alpha+1}}{\alpha+1} + C$ pri čemu je $u = 3-2x$, $\alpha = 7$.

b) $\int \frac{dx}{\cos^2(m-nx)} = \left| \begin{array}{l} d(m-nx) = -n\,dx \\ dx = \frac{-1}{n} d(m-nx) \end{array} \right| = -\frac{1}{n}\int \frac{d(m-nx)}{\cos^2(m-nx)} =$

$= -\frac{1}{n} tg(m-nx) + C$

Koristili smo formulu $\int \frac{du}{\cos^2 u} = tg\,u + C$ gdje $u = m-nx$.

c) $\int tg\varphi\, d\varphi = \int \frac{\sin\varphi}{\cos\varphi} d\varphi = \left| \begin{array}{l} d(\cos\varphi) = -\sin\varphi\, d\varphi \\ \sin\varphi\, d\varphi = -d(\cos\varphi) \end{array} \right| = -\int \frac{d(\cos\varphi)}{\cos\varphi}$

$= -\ln|\cos\varphi| + C$

Koristili smo formulu $\int \frac{du}{u} = \ln|u| + C$ gdje $u = \cos\varphi$.

Zadaci za vježbu

Odrediti sljedeće integrale

1. $\int x^4 dx$
2. $\int \sqrt[5]{t^2}\, dt$
3. $\int \frac{dy}{3y^2}$
4. $\int \frac{dx}{x+3}$
5. $\int (\alpha-5)^8 d\alpha$
6. $\int \frac{dx}{x^2+9}$
7. $\int \frac{dv}{\sqrt{v^2+7}}$
8. $\int \frac{dz}{2z^2-4}$
9. $\int \frac{dx}{\sqrt{4-x^2}}$
10. $\int \sin\frac{x}{3}\, dx$
11. $\int \frac{1}{\sin^2 2\varphi}\, d\varphi$
12. $\int e^{4x} dx$
13. $\int \frac{3\,dt}{5^{2t}}$
14. $\int \frac{dx}{2x+5}$
15. $\int \frac{dx}{(3x+2)^3}$
16. $\int \mathrm{ctg}\, x\, dx$

Rješenja:

1. $\frac{x^5}{5}$
2. $\frac{5}{7}\sqrt[5]{t^7}$
3. $-\frac{1}{3y}$
4. $\ln|x+3|$
5. $\frac{(\alpha-5)^9}{9}$
6. $\frac{1}{3}\,\mathrm{arctg}\frac{x}{3}$
7. $\ln(v+\sqrt{v^2+7})$
8. $\frac{1}{4\sqrt{2}}\ln\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right|$.
9. $\arcsin\frac{x}{2}$
10. $-3\cos\frac{x}{3}$
11. $-\frac{1}{2}\,\mathrm{ctg}\, 2\varphi$
12. $\frac{1}{4}e^{4x}$
13. $-\frac{3\cdot 5^{-2t}}{2\ln 5}$
14. $\frac{\ln|2x+5|}{2}$
15. $-\frac{1}{6(3x+2)^2}$
16. $\ln|\sin x|$

Izabrani **Zadaci za vježbu** sa rješenjima (iz lekcije Osnovne formule integriranja)

1. Pomoću osnovnih tabličnih integrala i najjednostavnijih pravila integracije odrediti sljedeće integrale:

a) $\int \sqrt{x}\, dx$

Rj. $\int \sqrt{x}\, dx = \int x^{\frac{1}{2}} dx = \dfrac{x^{\frac{1}{2}+1}}{\frac{3}{2}} + C = \frac{2}{3} x^{\frac{3}{2}} + C = \frac{2}{3}\sqrt{x^3} + C = \frac{2}{3} x\sqrt{x} + C$

b) $\int \sqrt[m]{x^n}\, dx$

Rj. $\int \sqrt[m]{x^n}\, dx = \int x^{\frac{n}{m}} dx = \dfrac{x^{\frac{n}{m}+1}}{\frac{m+n}{m}} + C = \dfrac{m}{m+n} x^{\frac{m+n}{m}} + C = \dfrac{m}{m+n}\sqrt{x^{m+n}} + C =$

$= \dfrac{m}{m+n}\sqrt[m]{x^m \cdot x^n} + C = \dfrac{m}{m+n} x \sqrt[m]{x^n}$

c) $\int \dfrac{dx}{x^2}$

Rj. $\int \dfrac{dx}{x^2} = \int x^{-2} dx = \dfrac{x^{-2+1}}{-1} + C = -x^{-1} + C = -\dfrac{1}{x} + C = C - \dfrac{1}{x}$

d) $\int 10^x dx$

Rj. $\int 10^x dx = \dfrac{10^x}{\ln 10} + C \approx 0{,}43429\ 10^x + C$

$\ln 10 \approx 2{,}30258$

e) $\int a^x e^x dx$

Rj. $\int a^x e^x dx = \int (a\cdot e)^x dx = \dfrac{(a\cdot e)^x}{\ln(a\cdot e)} + C = \dfrac{a^x e^x}{\ln a + \ln e} + C = \dfrac{a^x e^x}{1 + \ln a} + C$

f) $\int \frac{dx}{2\sqrt{x}}$

Rj: $\int \frac{dx}{2\sqrt{x}} = \frac{1}{2}\int x^{-\frac{1}{2}}dx = \frac{1}{2}\cdot\frac{x^{-\frac{1}{2}+1}}{\frac{1}{2}} + c = \frac{1}{2}\cdot 2\cdot x^{\frac{1}{2}} + c = \sqrt{x} + c$

g) $\int \frac{dh}{\sqrt{2gh}}$

Rj: $\int \frac{dh}{\sqrt{2gh}} = \int \frac{dh}{\sqrt{2g}\cdot\sqrt{h}} = \frac{1}{\sqrt{2g}}\int \frac{dh}{h^{\frac{1}{2}}} = \frac{1}{\sqrt{2g}}\int h^{-\frac{1}{2}}dh = \frac{1}{\sqrt{2g}}\cdot\frac{h^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{2}{\sqrt{2g}}\sqrt{h} + c = \sqrt{\frac{4h}{2g}} + c = \sqrt{\frac{2h}{g}} + c$

h) $\int 3{,}4\, x^{-0{,}17}dx$

Rj: $\int 3{,}4\, x^{-0{,}17}dx = 3{,}4\,\frac{x^{-0{,}17+1}}{0{,}83} + C = \frac{3{,}4}{0{,}83}\, x^{0{,}83} + c \approx 4{,}096\, x^{0{,}83} + c$

i) $\int (1-2u)\,du = \int du - 2\int u\,du$

Rj: $\int (1-2u)\,du = \int du - \int 2u\,du = u - 2\cdot\frac{u^2}{2} + c = u - u^2 + c$

j) $\int (\sqrt{x}+1)(x-\sqrt{x}+1)\,dx$

Rj: $\int (\sqrt{x}+1)(x-\sqrt{x}+1)\,dx = \int (x\sqrt{x} \underline{\underline{-x}} \underline{\underline{+\sqrt{x}}} \underline{\underline{+x}} \underline{\underline{-\sqrt{x}}} + 1)\,dx =$

$= \int (x^{\frac{3}{2}}+1)\,dx = \frac{x^{\frac{3}{2}+1}}{\frac{5}{2}} + x + c = \frac{2}{5}\sqrt{x^5} + x + c = \frac{2}{5}x^2\sqrt{x} + x + c$

k)^{v} $\int \frac{\sqrt{x} - x^3e^x + x^2}{x^3}\,dx$ Rj. $c - \frac{2}{3x\sqrt{x}} - e^x + \ln|x|$